SLOŽITOST

$f, g : \mathbb{N} \to \mathbb{R}^+$

$f \in O(g) \Leftrightarrow \exists c > 0 \quad \exists n_0 \quad \forall n > n_0 : f(n) \leq c \cdot g(n)$

$f \in \Omega(g) \Leftrightarrow \exists c > 0 \quad \exists n_0 \quad \forall n > n_0 : f(n) \geq c \cdot g(n)$

$f \in \Theta(g) \Leftrightarrow f \in O(g) \wedge f \in \Omega(g)$

1. $n^2 \in O(n^3)$

ANO: $c = 1 \quad \forall n \geq 1 \quad n^2 \leq n^3 \Leftrightarrow 1 \leq n$

2. $n^3 \in O(n^2)$

NE: $\forall c \quad \forall n_0 \quad \exists n > n_0 : f(n) > c \cdot g(n) \rightarrow n^3 > c \cdot n^2$

$n > c$

$\Rightarrow$ vezmu lib. $n > \max(c, n_0)$

3. $f \in O(g) \Rightarrow g \in O(f)$

NE: $f(n) = n^2, \; g(n) = n^3$

4. $f \in O(g) \Rightarrow g \in \Omega(f)$

víme: $f \in O(g) \Leftrightarrow \exists c \; \exists n_0 \; \forall n > n_0 \quad f(n) \leq c g(n)$

chceme: $g \in \Omega(f) \Leftrightarrow \exists d \; \exists n_0 \; \forall n > n_0 \quad g(n) \geq d f(n)$

$f(n) \leq \frac{1}{d} g(n)$

ANO: $d = \frac{1}{c}$

5. $f \in O(g) \vee g \in O(f)$

NE: $f(n) = \begin{cases} 1 & n \text{ sudé} \\ n & n \text{ liché} \end{cases} \qquad g(n) = \begin{cases} n & n \text{ sudé} \\ 1 & n \text{ liché} \end{cases}$

ukážeme $f \notin O(g) \Leftrightarrow \forall c, n_0 \; \exists n > n_0 : f(n) > c \cdot g(n)$

liché n: $f(n) = n, \; g(n) = 1 \qquad n > c \cdot 1$

$\rightarrow$ vezmu liché $n > \max(c, n_0)$